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Non-isothermal Couette flow of a rarefied gas between two rotating cylinders

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ABSTRACT. – An accurate numerical solution of the momentum and the heat transfer through a rarefied gas confined between two cylinders rotating with different angular velocities and having different temperatures has been obtained over a wide range of the Knudsen number on the basis of the Bhatnagar, Gross, Krook model equation. The viscous stress tensor, heat flux, and the fields of density, temperature and velocity are found. An analysis of the influence of the angular velocities and the temperature ratio on these quantities is given. © Elsevier, Paris.

Keywords. – Couette flow, heat transfer, rarefied gas.

1. Introduction

The problem of the isothermal cylindrical Couette flow of a rarefied gas has been investigated by many authors: (Cercignani and Sernagiotto, 1967), (Nanbu, 1984), (Sharipov and Kremer, 1996a), (Sharipov and Kremer, 1996b), (Soga and Oguchi, 1974), (Su and Springer, 1970), (Willis, 1962), (Willis, 1965), as well as that of heat conduction through a rarefied gas confined between two cylinders having different temperatures: (Willis, 1965), (Anderson, 1967), (Bassanini *et al.*, 1968), (Su and Willis, 1968), (Yu, 1970), (Lou and Shih, 1972), (Sone and Onishi, 1976), (Sharipov and Kremer, 1995a), (Sharipov and Kremer, 1995b). In the last two papers the heat flux through a rarefied gas between two rotating cylinders was calculated, but the angular velocities of the cylinders were assumed to be equal to each other and there was no momentum transfer.

In our previous papers (Sharipov and Kremer, 1995a), (Sharipov and Kremer, 1995b), (Sharipov and Kremer, 1996a), (Sharipov and Kremer, 1996b) the principal results based on the kinetic equation were: (i) the non-diagonal term of the viscous stress tensor is determined not only by the difference of angular velocities of the cylinders but by the values of both angular velocities; (ii) the diagonal term of this tensor is not equal to zero; (iii) the radial heat flux decreases if the cylinders rotate; and (iv) the radial temperature gradient causes a tangential heat flux between rotating cylinders. These results do not follow from the Navier-Stokes and Fourier equations of continuum mechanics.

The aim of the present paper is to obtain numerically a simultaneous transfer of momentum and heat between two rotating cylinders on the basis of the kinetic equation over a wide range of the gas rarefaction. This statement of the problem allows us to investigate the influence of the temperature difference on the momentum transfer (specially on the diagonal term of the stress tensor) as well as the influence of the angular velocity difference on the heat flux (specially on its tangential component). We shall consider a strong non-equilibrium state where the temperature difference is comparable with the average temperature and where the surface velocity of the cylinders is comparable with the sound velocity in the gas.

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Consider two coaxial cylinders with radii R_0, R_1 $(R_0 > R_1)$ confining a rarefied gas, with T_0 being the temperature of the outer cylinder, while T_1 is the temperature of the inner one. Moreover, the outer cylinder rotates with the angular velocity Ω_0, while the inner one has the angular velocity Ω_1. We are going to calculate the viscous stress tensor, the heat flux and the fields of density, bulk velocity and temperature between the cylinders.

2. Input equation

In terms of the kinetic theory the viscous stress tensor and the heat flux vector are defined via the velocity distribution function $f(t, \mathbf{r}', \mathbf{v})$ as

$$\sigma'_{\alpha\beta} = P\delta_{\alpha\beta} - m \int (v_\alpha - u'_\alpha)(v_\beta - u'_\beta) f d\mathbf{v}, \tag{1}$$

$$\mathbf{q}' = \int \frac{1}{2} m(\mathbf{v} - \mathbf{u}')(\mathbf{v} - \mathbf{u}')^2 f d\mathbf{v}, \tag{2}$$

respectively, where P is the gas pressure, $\delta_{\alpha\beta}$ is the Kronecker symbol, m is the molecular mass, $\mathbf{v}$ is the molecular velocity, $\mathbf{r}'$ denotes the spatial coordinates.

As an input equation for the distribution function $f(t, \mathbf{r}', \mathbf{v})$ we apply the Bhatnagar-Gross-Krook (BGK) model (Bhatnagar *et al.*, 1954) of the Boltzmann equation, which for a steady gas flow (the function f does not depend on the time t) in the cylindrical coordinates (r', φ, z') reads

$$v_r \frac{\partial f}{\partial r'} + \frac{v_\varphi}{r'} \frac{\partial f}{\partial \varphi} + v_z \frac{\partial f}{\partial z'} + \frac{v_\varphi^2}{r'} \frac{\partial f}{\partial v_r} - \frac{v_r v_\varphi}{r'} \frac{\partial f}{\partial v_\varphi} = \nu(f^M - f), \tag{3}$$

where f^M is the local Maxwellian

$$f^M(\mathbf{r}', \mathbf{v}) = n(\mathbf{r}') \left[\frac{m}{2\pi k T(\mathbf{r}')}\right]^{\frac{3}{2}} \exp\left\{-\frac{m[\mathbf{v} - \mathbf{u}'(\mathbf{r}')]^2}{2kT(\mathbf{r}')}\right\}, \tag{4}$$

and k is the Bolzmann constant. The number density n, bulk velocity $\mathbf{u}'$, and temperature T are defined via the distribution function as

$$n = \int f d\mathbf{v}, \quad \mathbf{u}' = \frac{1}{n} \int \mathbf{v} f d\mathbf{v}, \quad T = \frac{m}{3nk} \int (\mathbf{v} - \mathbf{u}')^2 f d\mathbf{v}. \tag{5}$$

The collision frequency ν can be chosen by two ways. One of them is to choose ν so as to obtain the correct expression for the shear viscosity η in the hydrodynamic regime, i.e.

$$\nu = \frac{P}{\eta}, \quad \eta = n\left(\frac{2mkT}{\pi}\right)^{\frac{1}{2}} \lambda, \tag{6}$$

where λ is the molecular mean-free-path. This expression for the frequency ν provides a correct description of the momentum transfer and was used by us in the papers (Sharipov and Kremer, 1996a), (Sharipov and Kremer, 1996b). Another way is to choose ν so as to obtain the correct expression for the thermal conductivity κ, i.e.

$$\nu = \frac{5}{2} \frac{Pk}{m\kappa}, \qquad \kappa = \frac{15k}{4m} \eta. \tag{7}$$

The last expression for ν provides a correct description of the heat transfer and was used by us in the paper (Sharipov and Kremer, 1995a). However, it is impossible to choose the collision frequency ν so as to obtain the correct expressions of the shear viscosity η and thermal conductivity κ simultaneously, because the BGK equation gives the Prandtl number equal to unity instead of the correct value 2/3. Below we shall use both expressions (6) and (7).

For the further derivations it is convenient to introduce the following dimensionless quantities

$$r = \frac{r'}{R_0}, \quad \mathbf{c} = \beta^{1/2}\mathbf{v}, \quad \mathbf{u} = \beta^{1/2}\mathbf{u}', \quad v = \frac{n}{n_0}, \quad \tau = \frac{T}{T_0},$$
$$\phi = \frac{f}{n_0\beta^{3/2}}, \quad \omega_0 = \beta^{1/2}R_0\Omega_0, \quad \omega_1 = \beta^{1/2}R_0\Omega_1,$$
$$\sigma_{\alpha\beta} = \frac{\beta}{mn_0}\sigma'_{\alpha\beta}, \quad \mathbf{q} = \frac{2\beta^{3/2}}{n_0 m}\mathbf{q}', \quad \beta = \frac{m}{2kT_0},$$

where n_0 is the number density in the state of equilibrium when $T_1 = T_0$ and $\Omega_1 = \Omega_0 = 0$. Then, regarding the axial symmetry of the gas flow the BGK equation (3) can be presented in the dimensionless variables as

$$c_r\frac{\partial\phi}{\partial r} - \frac{c_\varphi}{r}\frac{\partial\phi}{\partial\theta} = \delta v\sqrt{\tau}(\phi^M - \phi), \tag{8}$$

where $\theta = \arctan(c_\varphi/c_r)$ and

$$\phi^M = \frac{v}{(\pi\tau)^{3/2}}\exp\left[-\frac{c_r^2 + (c_\varphi - u_\varphi)^2 + c_z^2}{\tau}\right]. \tag{9}$$

The expression for the rarefaction parameter δ depends on the choice of the collision frequency ν. If Eq. (6) is used for ν we have

$$\delta = \frac{\sqrt{\pi}}{2}\frac{R_0}{\lambda_0}. \tag{10}$$

Using Eq. (7) we obtain

$$\delta = \frac{\sqrt{\pi}}{3}\frac{R_0}{\lambda_0}, \tag{11}$$

where λ_0 is the mean-free-path in the state of equilibrium. One can see that in solving Eq. (8) we shall find the viscous stress tensor and the heat flux as a function of the rarefaction parameter δ. Thus, to compare the present result with another obtained on the basis of the BGK equation we do not need to indicate whether expression (6) or (7) is used for the collision frequency. The expression for ν is necessary only if one needs to relate the solution of the kinetic equation to the Knudsen number $Kn = \lambda_0/R_0$. For this purpose it is better to use Eq. (6) for ν or Eq. (10) for δ if one needs to know the velocity profile and viscous stress tensor as a function of the Knudsen number. If one needs to relate the temperature field and the heat flux to the Knudsen number, it is better to use Eq. (7) for ν or Eq. (11) for δ.

We assume diffuse reflection of the particles by the walls, i.e.

$$\phi = \frac{v_{\omega 0}}{\pi^{3/2}}\exp[-c_r^2 - (c_\varphi - \omega_0)^2 - c_z^2], \qquad \text{at} \quad r = 1, \quad c_r < 0, \tag{12}$$

$$\phi = \frac{v_{\omega 1}}{(\pi T_1/T_0)^{3/2}}\exp\left[-\frac{c_r^2 + (c_\varphi - \omega_1 R_1/R_0)^2 + c_z^2}{T_1/T_0}\right] \qquad \text{at} \quad r = \frac{R_1}{R_0}, \quad c_r > 0, \tag{13}$$

where the quantities $v_{\omega 0}$ and $v_{\omega 1}$ are calculated from the impenetrability condition on the walls of the cylinders.

3. Method of solution

Equation (8) with the boundary condition (12) and (13) has been solved by the discrete velocity method. Since the method is described in details in the papers (Sharipov and Kremer, 1995a), (Sharipov and Kremer, 1995b), (Sharipov and Kremer, 1996a), (Sharipov and Kremer, 1996b), here we do not need to describe it. We shall note only that an improvement in the method has been made.

A typical feature of the rarefied gas flow around a convex body is the discontinuity of the distribution function. As was indicated by Sugimoto and Sone (1992) the difference scheme of the discrete velocity method (see e.g. Eq. (23) in the paper (Sharipov and Kremer, 1996b)) is not appropriate to the discontinuity of the distribution function around the inner cylinder. In other words, it is impossible to reduce significantly the numerical error using the ordinary difference scheme, even if the grid increments are very small. Since the scheme improvement taking the discontinuity into account is described in detail in the work by Sugimoto and Sone (1992), we shall outline only the main features here.

At points of discontinuity, two values of the distribution function are calculated by integrating Eq. (8) along the characteristics: one characteristic begins on the inner cylinder yielding the first value, while the other begins on the outer cylinder and yields a second value of the distribution function. We then use these values to calculate the distribution function at the grid points nearest to the discontinuity: the first value is used for points placed between the inner cylinder and the discontinuity point, while the second value is used for grid points which are external relative to the discontinuity.

Moreover, to eliminate the roundoff error all real numbers in the numerical program were calculated with double precision.

4. Results and discussions

The calculations have been carried out for $R_1/R_0 = 0.5$ and $T_1/T_0 = 2$ over the range of δ from 0.04 to 20. The angular velocities were varied so that their difference was fixed at $\omega_0 - \omega_1 = 1$. Three values of ω_0 have

TABLE I. – Viscous stress tensor $\sigma_{r\varphi}$ and radial heat flux q_r in the midpoint vs δ and ω_0.

		$\sigma_{r\varphi}$		q_r	
δ	ω_0	present work	Sh. & K., 1996a $T_1 = T_0$	present work	Sh. & K., 1995a $\omega_1 = \omega_0$
0.04	0.0	0.06778	0.06380	0.4493	...
	0.5	0.06100	0.05726	0.4053	...
	1.0	0.04414	0.04178	0.2938	...
0.1	0.0	0.06765	0.06357	0.4466	0.4022
	0.5	0.06095	0.05717	0.4034	0.3633
	1.0	0.04413	0.04195	0.2930	0.2644
0.2	0.0	0.06736	0.06318	0.4418	0.3981
	0.5	0.06083	0.05714	0.4000	0.3600
	1.0	0.04410	0.04183	0.2917	0.2630
0.4	0.0	0.06666	0.06238	0.4315	0.3893
	0.5	0.06051	0.05659	0.3927	0.3532
	1.0	0.04403	0.04160	0.2890	0.2600
1.0	0.0	0.06418	0.05993	0.4007	0.3627
	0.5	0.05918	0.05495	0.3702	0.3327
	1.0	0.04371	0.04094	0.2804	0.2508
2.0	0.0	0.06058	0.05592	0.3588	0.3241
	0.5	0.05695	0.05216	0.3383	0.3016
	1.0	0.04329	0.03973	0.2684	0.2360
4.0	0.0	0.05314	0.04876	0.2920	0.2658
	0.5	0.05138	0.04670	0.2824	0.2524
	1.0	0.04127	0.03723	0.2397	0.2091
10.	0.0	0.03818	0.03412	0.1864	0.1724
	0.5	0.03812	0.03388	0.1850	0.1678
	1.0	0.03375	0.02976	0.1722	0.1512
20.	0.0	0.02560	0.02206	0.1159	0.1084
	0.5	0.02586	0.02231	0.1160	0.1067
	1.0	0.02450	0.02100	0.1125	0.1008

been taken: $\omega_0 = 0$ (the outer cylinder is at rest, the inner one rotates), $\omega_0 = 0.5$ (the cylinders rotate in the opposite directions) and $\omega_0 = 1$ (the outer cylinder rotates, the inner cylinder is at rest).

Test calculations carried out for $\delta = 0.1$, 1 and 20 with a doubling of every grid parameter showed that the non-diagonal element of the viscous stress $\sigma_{r\varphi}$ and the radial heat flux q_r change by less than 0.5% for $\delta \leq 1$ and 1% for $\delta > 1$.

To provide one more criterion for the calculations, the fulfillment of the conservation laws of momentum ($\sigma_{r\varphi}r^2 = \mathrm{const.}$) and of energy ($(q_r - 2u_\varphi\sigma_{r\varphi})r = \mathrm{const.}$), were verified in all calculations. An analysis of the numerical data shows that the distortion of the conservation laws does not exceed 0.5% for $\delta \leq 1$ and 1% for $\delta > 1$. Thus, it can be considered that the relative numerical error of the quantities $\sigma_{r\varphi}$ and q_r is 0.5% for $\delta \leq 1$ and 1% for $\delta > 1$.

The other quantities, such as the diagonal element of the viscous stress $\sigma_{\varphi\varphi}$ and the tangential heat flux q_φ have a larger relative numerical error, because their absolute values are essentially smaller than $\sigma_{r\varphi}$ and q_r, respectively. For instance, the ratio $\sigma_{\varphi\varphi}/\sigma_{r\varphi}$ is of order about 0.1 for all values of δ. The ratio q_φ/q_r is of order about 0.5 for $\delta \leq 1$ and of order about 0.1 for $\delta > 1$. The numerical error of $\sigma_{\varphi\varphi}$ and q_φ varies from 1% for $\delta \leq 1$ to 4% for $\delta > 1$. It would be possible to reduce the last error to 1% using a denser grid, but the computation time increases so drastically that the slight correction in the results would be very costly.

In Table I the values of the viscous stress tensor $\sigma_{r\varphi}$ (third column) at the midpoint ($r = 0.75$) between the cylinders are presented. One can see that the viscous stress tensor $\sigma_{r\varphi}$ is determined not only by the angular velocity difference ($\omega_0 - \omega_1$) but by the angular velocity ω_0 too. The influence of ω_0 on $\sigma_{r\varphi}$ decreases with increasing δ. To compare the present results for $\sigma_{r\varphi}$with those for the isothermal Couette flow ($T_1 = T_0$) we give the results of Sharipov and Kremer, (1996b) in the fourth column. It can be seen that the difference between the viscous stress tensors for the non-isothermal flow and for the isothermal one reaches 15%.

The values of the radial heat flux q_r at the midpoint are also presented in Table I (fifth column). One can see that the heat flux depends significantly on the angular velocity ω_0. To compare the values of the heat flux q_r with those obtained for the cylinders rotating with the equal velocities ($\omega_0 = \omega_1$) the results of Sharipov

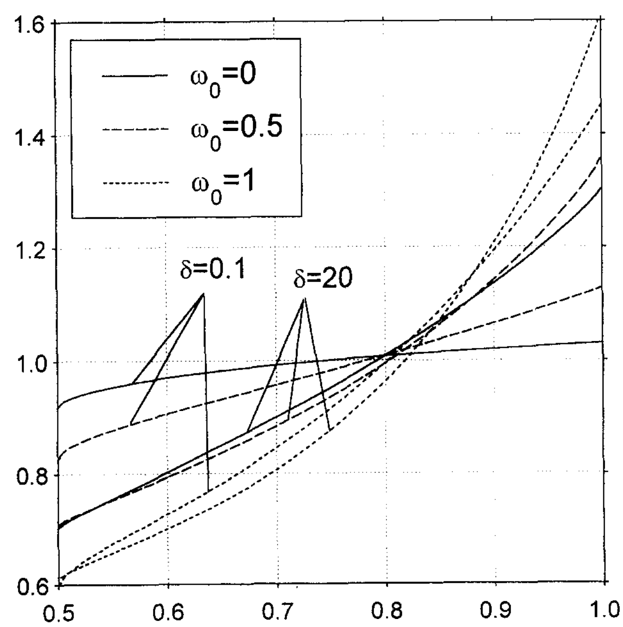

Fig. 1. – Number density ν vs r.

and Kremer, (1995a) are given in the sixth column. It can be seen that the heat flux for the different angular velocities essentially exceeds the heat flux for the equal velocities. The difference is again about 15%.

In Figures 1-5 the distributions of the number density υ, the temperature τ, the bulk velocity $(u_\varphi - \omega_0 r)$, the diagonal component of the viscous stress tensor $\sigma_{\varphi\varphi}$ and the tangential heat flux q_φ are presented. It can be

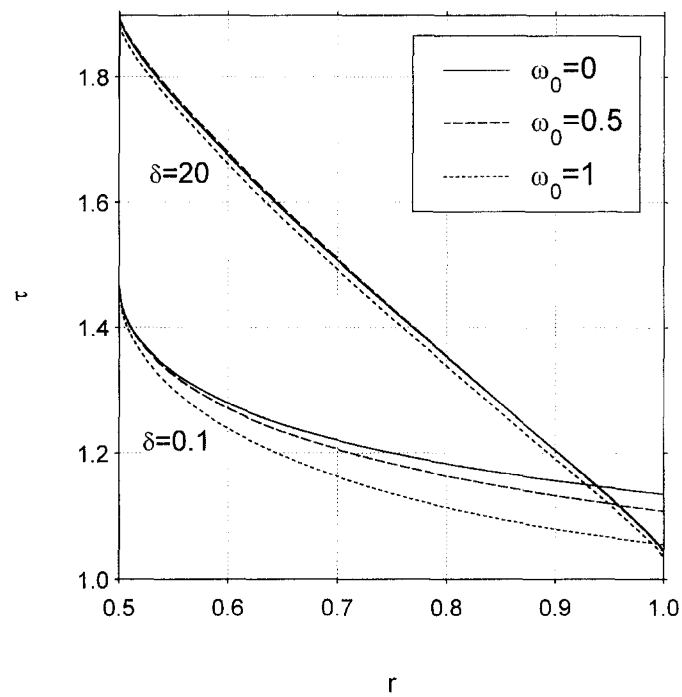

Fig. 2. – Temperature τ vs r.

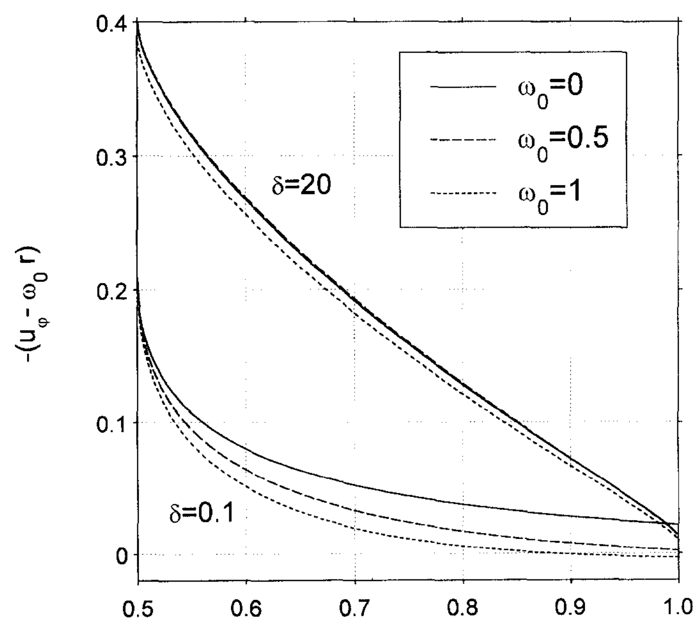

Fig. 3. – Bulk velocity $(u_\varphi - \omega_0 r)$ vs r.

seen that the number density v (Figure 1) is strongly affected by the angular velocity ω_0. If one compares the density distribution v with that obtained by Sharipov and Kremer, (1995a) for the equal velocities $(\omega_0 = \omega_1)$

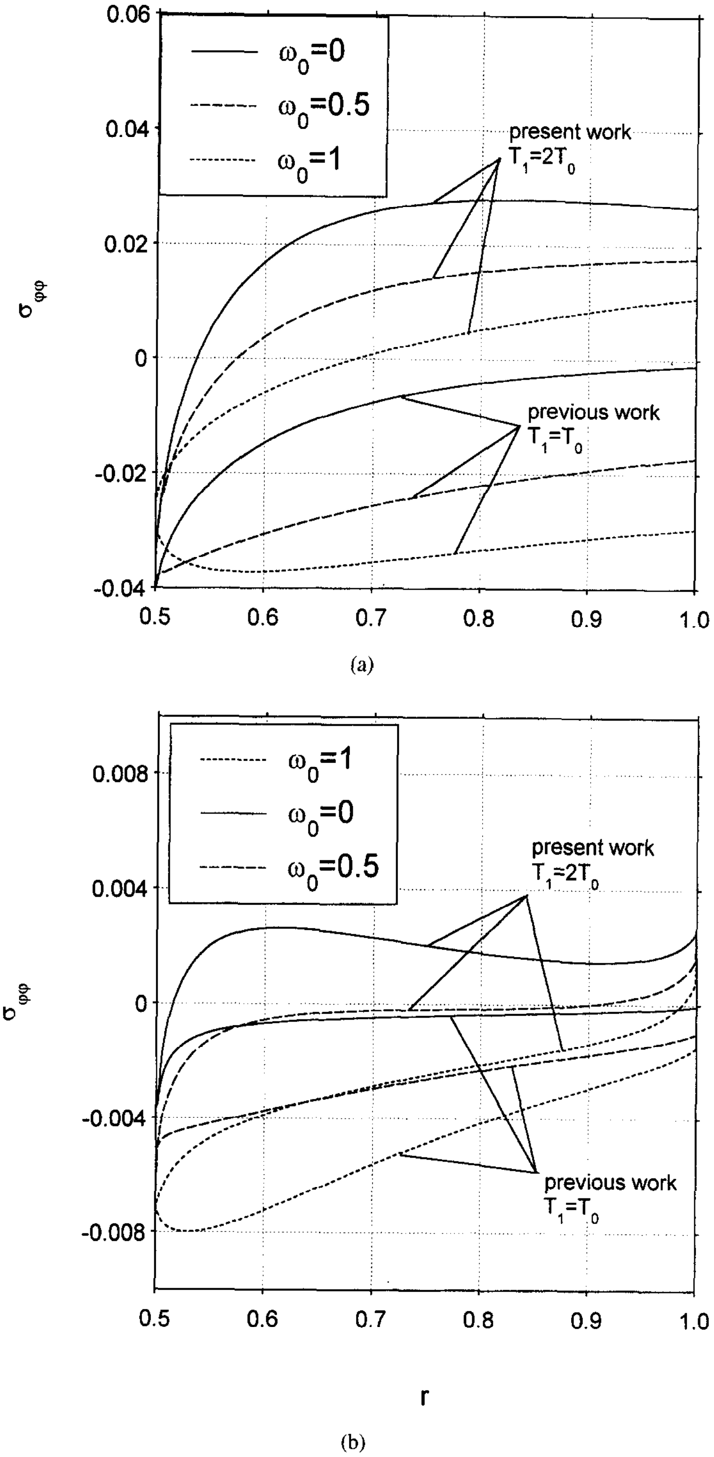

Fig. 4. – Viscous stress tensor $\sigma_{\varphi\varphi}$ vs r: (a) $\delta = 0.04$; (b) $\delta = 20$.

and for the same temperature ratio $T_1/T_0 = 2$ one finds that the difference is small. So, the dependence of the number density v on the angular velocity of the inner cylinder ω_1 is weak.

The temperature distribution τ (Figure 2) obtained here differs very slightly from the distribution obtained for equal velocities $\omega_1 = \omega_0$ by Sharipov and Kremer, (1995a). From this comparison we conclude that the temperature distribution is not affected by the angular velocities ω_0 and ω_1 and depends on the temperature of the cylinders and on the rarefaction parameter δ.

The velocity profiles $(u_\varphi - \omega_0 r)$ (Figure 3) in the problem in question differ very slightly from those obtained for the equal temperatures $(T_1 = T_0)$ in Sharipov and Kremer, (1996b). This comparison allows us to make the conclusion: the velocity profile is not affected by the temperatures of the cylinders, but depends only on the velocity difference $(\omega_1 - \omega_0)$ and on the rarefaction parameter δ.

A comparison between the diagonal term of the viscous stress $\sigma_{\varphi\varphi}$ obtained here with that obtained for the isothermal Couette flow $(T_1 = T_0)$ is given in Figure 4. One can see that the values of $\sigma_{\varphi\varphi}$ for the non-isothermal flow are different from those for the isothermal flow. It can be said that the temperature difference $(T_1 - T_0)$ creates a shift in the values of $\sigma_{\varphi\varphi}$.

The profile of the tangential heat flux q_φ is given in Figure 5. If one compares this figure with figure 1 in Sharipov and Kremer, (1995a), one finds that the behavior of the tangential heat flux q_φ obtained here differs completely from that obtained for the equal angular velocities $(\omega_0 = \omega_1)$. First of all, the values of q_φ obtained here are negative, while the values of q_φ obtained for the equal velocities $(\omega_0 = \omega_1)$ are positive. So, we may conclude that the quantity q_φ is determined by the velocity difference $(\omega_1 - \omega_0)$ as well as by the temperature difference $(T_1 - T_0)$.

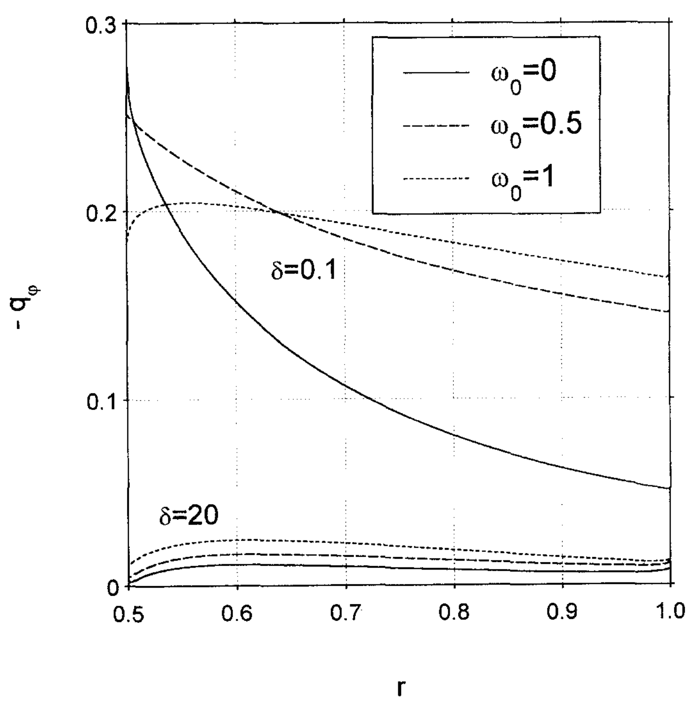

Fig. 5. – Heat flux q_φ vs r.

5. Conclusions

The viscous stress tensor, heat flux, and the fields of number density, temperature, bulk velocity in a rarefied gas confined between two cylinders rotating with different angular velocities and having different temperatures have been calculated from the kinetic BGK equation over a wide range of the Knudsen number.

The analysis of the numerical data has shown that:

(i) The viscous stress tensor, beside the angular velocities ω_0 and ω_1, is determined by the temperature difference $(T_1 - T_0)$ too. The values of its non-diagonal component for $T_1/T_0 = 2$ exceed those obtained for the isothermal flow $(T_1 = T_0)$ by up to 15%. The values of the diagonal components of the stress tensor have a shift in comparison with the values obtained for the isothermal flow.

(ii) The heat flux, beside the temperature difference $(T_1 - T_0)$, is determined by the angular velocities ω_0 and ω_1. The values of the radial heat flux for $\omega_0 - \omega_1 = 1$ exceed those obtained for the equal velocities $(\omega_0 = \omega_1)$ by up to 15%. The tangential heat flux in the case of the different angular velocities has a quite different profile in comparison with that obtained for the equal velocities.

(iii) The temperature distribution is not affected by the angular velocity difference $(\omega_0 - \omega_1)$ and determined only by the temperatures of the cylinders and by the rarefaction parameter.

(iv) The velocity profile is not affected by the temperature difference $(T_1 - T_0)$ and determined only by the angular velocity difference $(\omega_0 - \omega_1)$ and by the rarefaction parameter.

(v) The number density distribution is determined by the temperature ratio T_1/T_0 and by the angular velocity of the outer cylinder ω_0

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REFERENCES

ANDERSON D.G.M., 1967, On the steady Krook kinetic equation: Part 2. *J. Plasma Physics*, **1**, 255–265.

BASSANINI P., CERCIGNANI C., PAGANI C.D., 1968, Influence of the accomodation coefficient on the heat transfer in a rarefied gas, *Int. J. Heat and Mass Transfer*, **11**, 1359–1369.

BHATNAGAR P.L., GROSS E. P., KROOK M., 1954, A model for collision processes in gases, *Phys. Rev.*, **94**, 511–525.

CERCIGNANI C., SERNAGIOTTO F., 1967, Cylindrical Couette flow of a rarefied gas, *Phys. Fluid*, **10**, 1200–1204.

LOU Y.S., and SHIH T.K., 1972, Nonlinear heat conduction in rarefied gases confined between concentric cylinders and spheres, *Phys. Fluids*, **15**, 785–788.

NANBU K., 1984, Analysis of cylindrical Couette flows by use of the direction simulation method, *Phys. Fluids*, **27**, 2632–2635.

SHARIPOV F.M., KREMER G.M., 1995a, On the frame dependence of constitutive equations. I. Heat transfer through a rarefied gas between two rotating cylinders, *Continuum Mech. Thermod.*, **7**, 57–72.

SHARIPOV F.M., and KREMER G.M., 1995b, Heat conduction through a rarefied gas between two rotating cylinders at small temperature difference, *J. Appl. Phys. Math. (ZAMP)*, **46**, 680–692.

SHARIPOV F.M., and KREMER G.M., 1996a, Linear Couette flow between two rotating cylinders, *Eur. J. Mech. B/Fluids*, **15**, 493–505.

SHARIPOV F.M., and KREMER G.M., 1996b, Nonlinear Couette flow between two rotating cylinders, *Tran. Theory Stat. Mech.*, **25**, 217–229.

SOGA T., OGUCHI H., 1974, A numerical analysis of nonlinear cylindrical Couette flow. In *Rarefied Gas Dynamics*, Proc. 9th Int. Symp. (DFVLR Press, Porz-Wahn, Germany, 1974), Paper No. A.17.

SONE Y., ONISHI Y., 1976, Heat transfer through a rarefied gas between coaxial circular cylinders with large temperature difference. In *Rarefied Gas Dynamics*, Proc. 10th Int. Symp. (Aspen, USA), pp. 295–310.

SU Ch.-L., SPRINGER R.W., 1970, Modified discrete ordinate approach in rarefied gas dynamics, *Phys. Fluids*, **13**, 1637–1639.

SU C.L., WILLIS D.R., 1968, Heat conduction in rarefied gas between concentric cylinders, *Phys. Fluids*, **11**, 2131–2143.

SUGIMOTO H., SONE Y., 1992, Numerical analysis of steady flows of a gas evaporating from its cylindrical condensed phase on the basis of kinetic theory, *Phys. Fluids A*, **4**, 419–440.

WILLIS D.R., 1962, Comparison of kinetic theory analyses of linearized Couette flow, *Phys. Fluids*, **5**, 127–135.
WILLIS D.R., 1965, Heat transfer and shear between coaxial cylinders for large Knudsen numbers, *Phys. Fluids*, **8**, 1908–1910.
YU E.Y., 1970, Heat conduction between concentric cylinders in the slip regime, *Phys. Fluids*, **13**, 2484–2491.

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